

Synchronization of Mutually Versus Unidirectionally Coupled Chaotic Semiconductor Lasers

Noam Gross¹, Wolfgang Kinzel², Ido Kanter¹, Michael Rosenbluh¹, Lev Khaykovich¹

¹*Department of Physics, Bar-Ilan University, Ramat-Gan, 52900 Israel, and*

²*Institut für Theoretische Physik, Universität Würzburg, Am Hubland 97074 Würzburg, Germany*

Synchronization dynamics of mutually coupled chaotic semiconductor lasers are investigated experimentally and compared to identical synchronization of unidirectionally coupled lasers. Mutual coupling shows high quality synchronization in a broad range of self-feedback and coupling strengths. It is found to be tolerant to significant parameter mismatch which for unidirectional coupling would result in loss of synchronization. The advantages of mutual coupling are emphasized in light of its potential use in chaos communications.

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Communication via chaotic signals has drawn much attention in the last two decades. A semiconductor laser subjected to an external feedback displays complex chaotic behavior [1, 2, 3, 4]. Two chaotic lasers have been shown to synchronize with each other and are excellent candidates for fast and secure communication [5, 6, 7]. Recently, a field experiment using long fiber spans of commercial optical networks has been conducted, in which chaotic optical communications at high transmission and low bit-error rates were reported [6]. In this experiment, a receiver laser was synchronized to a transmitter laser unidirectionally allowing unidirectional information flow only. The advantage of a mutually synchronized system is that it increases the efficiency of the apparatus by allowing a bilateral conversation. Moreover unidirectional communication is a private-key system where the system parameters serve as the secret key. In ref. [8] it was shown that it might be possible to use the synchronization of two mutually coupled symmetric chaotic systems in a novel cryptographic key-exchange protocol, whereby secret messages can be transmitted over public channels without using any previous secrets. Recently we made a first step toward realization of such protocol by suggesting a mutual chaos pass filter procedure based on experiments which revealed a window of parameters where mutual coupling is advantageous over its unidirectional counterpart [9]. Here we explore the robustness of mutually coupled lasers to different experimental parameters.

Chaos synchronization via unidirectional optical coupling is of two types: identical and generalized [5, 10, 11, 12]. Identical, also known as anticipated synchronization, appears when two nearly identical lasers are subjected to the same optical feedback. One laser simply reproduces the dynamics of the other. The most simple and effective way to achieve this type of synchronization is by setting the transmitter laser's (*TL*) external feedback strength, κ_t , and the receiver laser's (*RL*) coupling strength, σ_r , to be equal to each other, $\kappa_t = \sigma_r$, while the *RL* external feedback, $\kappa_r = 0$ [5]. Generalized synchronization requires strong injection (coupling strength) and the *RL* behaves more like a driven oscillator with its

output driven by the injection signal [13]. Such synchronization is robust to channel disturbances [12] and works well even under non identical laser parameters, hence it is favorable for chaos communication [6, 7, 11, 15, 16]. Although identical synchronization can achieve higher fidelity, it is difficult to realize in real systems due to its high sensitivity to laser parameter mismatch [12, 13, 14].

Mutual optical coupling has been explored mostly in a face to face configuration, in which the lasers have no external cavities, or self feedback [17, 18, 19]. In this case, achronal synchronization was found, for which one of the lasers is a leader and the other a laggard in a time dependent manner, thus assigning asymmetric physical roles to the lasers, even under symmetric operating conditions. Recently, we introduced mutual optical coupling with the addition of self feedback and observed isochronal synchronization [20], i.e. the lasers are synchronized with zero time delay. A necessary condition for this type of synchronization is that the self feedback round-trip lengths of both lasers be equal to the coupling length (the distance between them). When both lasers are subjected to approximately the same feedback, high quality identical synchronization is achieved.

In this paper we focus on isochronal synchronization of mutually coupled lasers. We explore the feedback and coupling strength phase-space and show that this type of identical synchronization has much higher tolerance for parameter mismatch than its unidirectional counterpart. Thus, not only does mutual coupling allow a bidirectional flow of information, it also combines the high correlation offered by identical synchronization with the robustness typical of generalized synchronization.

Our experimental setup is depicted in Fig.1(a) and in Fig.1(b) for mutual and unidirectional coupling, respectively. We use two single-mode lasers emitting at 660 nm and operating close to their threshold. The lasers are "off-the-shelf" non-preselected devices, however, at room temperature they emit at nearly identical optical wave length ($\Delta\lambda = 0.2\text{nm}$) and have the same threshold current (43.1mA) and P/I curve (better than 99% match). The temperature of each laser is stabilized to better than 0.01K and the absolute temperature of each laser is ad-

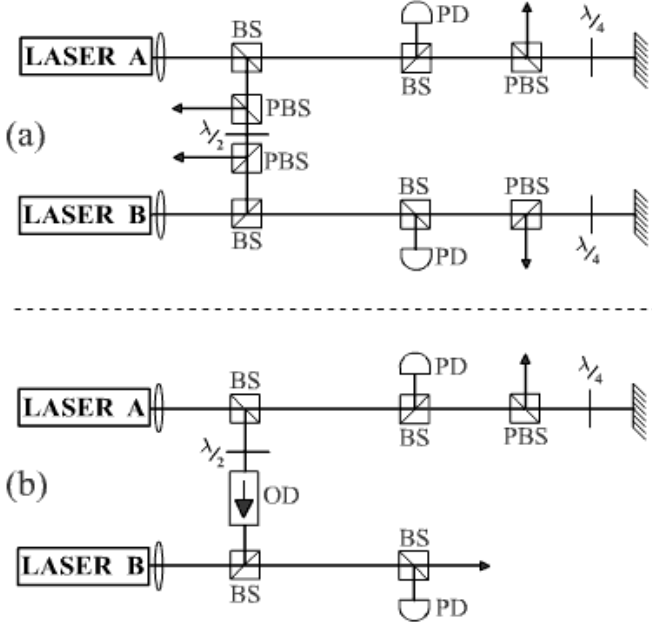


Figure 1: Schematic of two coupled lasers for mutual (a) and unidirectional (b) synchronization. BS - Beam Splitter; PBS - Polarizing Beam Splitter; OD - Optical Isolator; PD - Photodetector

justed so as to match the individual laser wavelengths to be identical. The length of the external cavities is set to 180 cm (round trip time 12 ns). Self feedback strength is adjusted using a $\lambda/4$ wave plate and a polarizing beam splitter. In the mutual coupling experiment (Fig.1(a)), the two lasers (A and B) are mutually coupled by injecting a fraction of each one's output power to the other. Coupling power is adjusted using a $\lambda/2$ wave plate and two polarizing beam splitters. In the unidirectional coupling experiment (Fig.1(b)), laser B is coupled unidirectionally to laser A, with unidirectionality ensured by an optical diode (-34 dB). Coupling power is adjusted using a $\lambda/2$ wave plate located in front of the optical isolator. In both cases, coupling optical paths are set to be equal the self feedback round trip path. Two fast photodetectors (response time < 500 ps) are used to monitor the laser intensities which are simultaneously recorded with a digital oscilloscope (500MHz, 1GS/s).

The feedback strength of a laser is measured via the reduction of the laser's threshold current. The relation between the threshold current, I_{th} , of a laser with external feedback and the threshold current a solitary laser, I_{th}^{sol} , is [21]

$$\frac{I_{th}}{I_{th}^{sol}} = 1 - \gamma \ln(1 + \delta \frac{\kappa}{\kappa_0}) \quad (1)$$

where γ and δ are constants, κ is the feedback strength and κ_0 is the feedback strength equivalent to a reduction

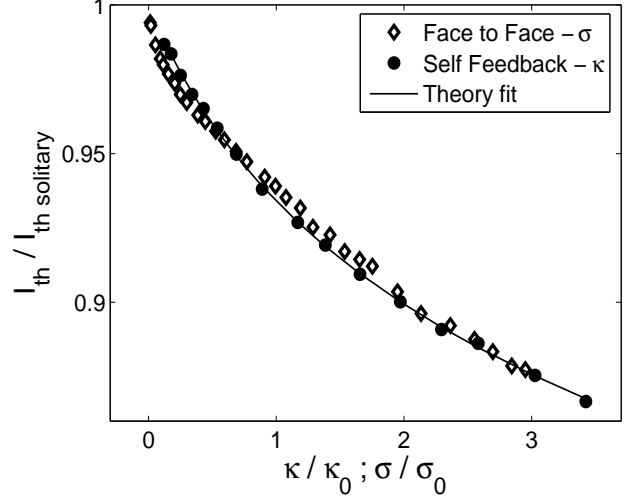


Figure 2: Measured feedback strength versus the reduction in laser threshold current. σ_0 and κ_0 are the feedback strength values required for a 6.6% reduction in I_{th}^{sol} . Theoretical fit is from eq.(1) (see text).

of 6.6% in I_{th}^{sol} . κ is proportional to the fraction of light power coupled back into the cavity. In Fig.2 this relation is confirmed experimentally (closed circles). The theoretical fit provides the values $\gamma = 0.0754$ and $\delta = 1.40$. We also find that the feedback strength between two coupled lasers behaves in agreement with eq.(1). To measure the influence of the coupling strength on the laser's threshold current, the lasers were set in a face to face configuration, in which they are exposed to mutual feedback without having an external cavity of their own. Laser wavelengths were set by temperature control to achieve a maximum overlap between individual laser modes. This measurement is also shown in Fig.2 as open diamonds. Similar to self feedback, σ_0 is equivalent to a 6.6% reduction in I_{th}^{sol} of both lasers, and σ is the coupling strength. We attribute the small differences between the self and mutual feedbacks due to the residual difference in lasers frequencies in the case of mutual feedback.

The phase-space of when good synchronization is achieved as a function of self feedback (κ) and mutual coupling strength (σ) is shown in Fig.3. At any point in this phase space, symmetry between the two lasers is maintained so that $\kappa = \kappa_A = \kappa_B$ and $\sigma = \sigma_A = \sigma_B$. The quality of synchronization between lasers is evaluated by the correlation coefficient [20], ρ defined as follows:

$$\rho = \frac{\sum^i (I_A^i - \langle I_A^i \rangle) \cdot (I_B^i - \langle I_B^i \rangle)}{\sqrt{\sum^i (I_A^i - \langle I_A^i \rangle)^2 \cdot \sum^i (I_B^i - \langle I_B^i \rangle)^2}} \quad (2)$$

where I_A^i and I_B^i are the instantaneous intensities of lasers A and B respectively (see note [22]). The best synchronization ($\rho \geq 0.91$) is achieved for feedback values of κ_0 and σ_0 . The synchronization, however, is ro-

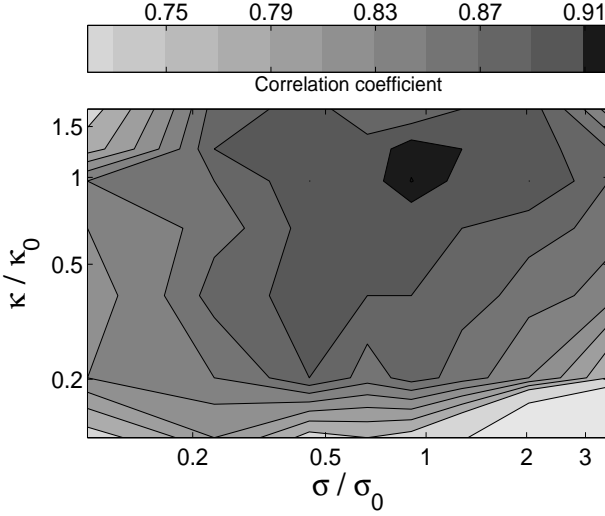


Figure 3: Mutual coupling phase space where the correlation coefficient (gray level) is plotted as a function of κ and σ .

bust to large changes in feedback strength and stable isochronal synchronization is maintained even for a 50% deviation from these values. This robustness is very encouraging considering real communication applications where channel intensity (and thus feedback strength) might suffer from fluctuations and disturbances. A theoretical phase-space, based on a numerical solution of the Lang-Kobayashi equations is available in ref. [20] and confirms the existence of a broad area where the lasers exhibit stable synchronization. When considering the case of $\kappa = \sigma$, stable synchronization is intuitively understood since each laser is subjected to a feedback consisting of equal contributions from both laser intensities, hence their "driving force" is identical. Our measurements, however, show that $\kappa = \sigma$ is not necessary and the coupling intensity can be significantly weakened without destroying the correlation, thus allowing for conditions favorable for the realization of secure information exchange [8, 9].

The unidirectional coupling phase space is presented in Fig.4. Here, the *TL* (laser *A*) is subjected only to a self feedback from its external cavity (κ_A) and the *RL* (laser *B*) is subjected only to coupling from laser *A* (σ_B), while the condition of a total feedback strength symmetry is maintained ($\kappa_A = \sigma_B$). As in mutual coupling, good synchronization is achieved in the vicinity of κ_0 and σ_0 . For $\kappa_A \neq \sigma_B$ the correlation drops rapidly and therefore we examined an additional self feedback to the *RL*, also known as a closed loop scheme [7]. However, all attempts resulted in deterioration of the synchronization. In particular, for the closed loop case, synchronization is very sensitive to the phase of the returning field, demanding a careful adjustment of the external cavity length to sub wavelength levels [7, 23]. We note that we have not observed a phase dependence in the configuration of mutually coupled lasers.

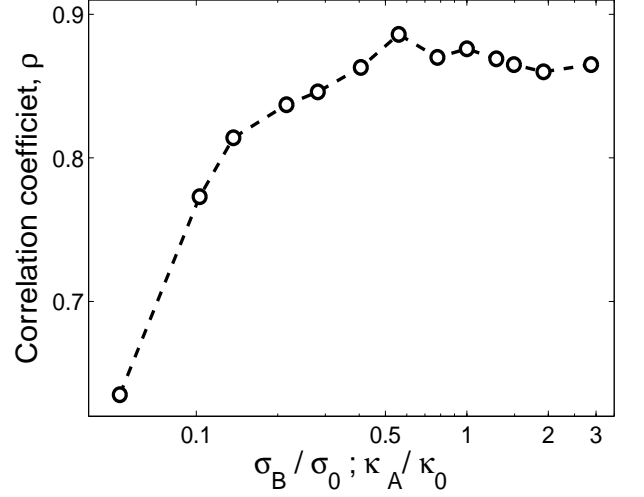


Figure 4: Correlation coefficient as a function of κ_A and σ_B for unidirectional coupling. $\kappa_A = \sigma_B$ in all cases.

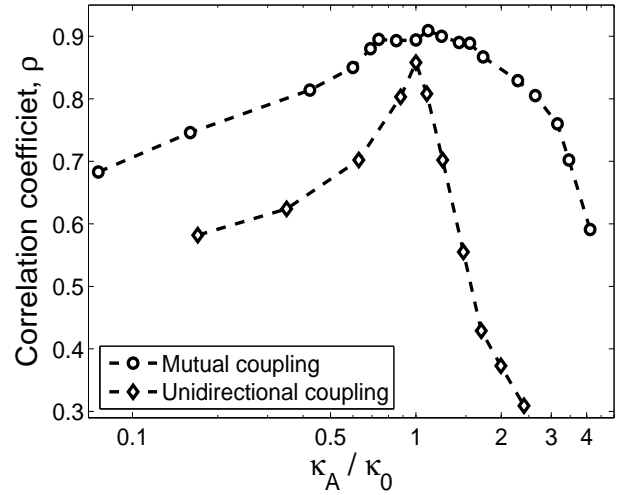


Figure 5: Correlation coefficient for different values of κ_A , laser *A*'s self feedback rate. Coupling strength is σ_0 . For mutual coupling (circles) $\kappa_b = \kappa_0$ and $\kappa_b = 0$ for unidirectional coupling (diamonds).

In Fig.5 we demonstrate the sensitivity to self feedback strength for both unidirectional and mutual coupling configurations. Correlation coefficient is calculated for different values of κ_A , while $\kappa_b = \kappa_0$ for mutual coupling and $\kappa_b = 0$ for unidirectional coupling. In either cases Coupling strength is σ_0 . For unidirectional coupling (diamonds), good synchronization cannot be attained unless $\kappa_A = \kappa_0$. However, for mutual coupling (circles), small deviations of the self feedback strength of one laser with respect to that of the other, does not affect synchronization quality. In this case, good synchronization of $\rho = 0.9$ is obtained for self feedback range of $0.7\kappa_0 \leq \kappa_A \leq 1.5\kappa_0$.

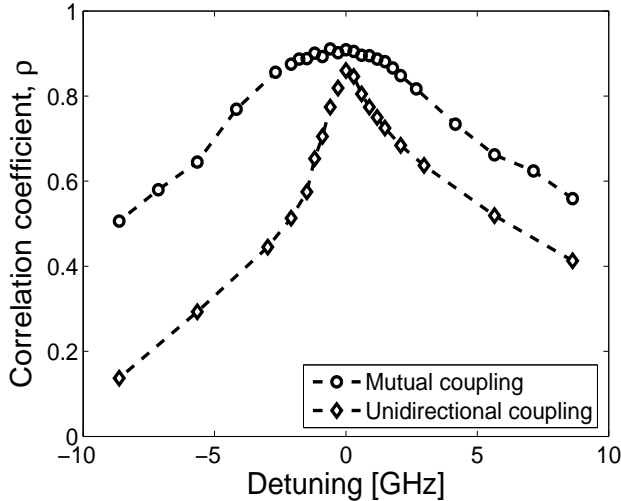


Figure 6: Correlation coefficient as a function of detuning of laser *A*'s optical frequency with respect to laser *B*, for mutual (circles) and unidirectional (diamonds) coupling. Feedback strengths are σ_0 and κ_0 .

Thus mutual coupling exhibits high tolerance for asymmetry between the operating lasers, while unidirectional coupling is very sensitive to the mismatch and synchronization appears only if the condition of total feedback symmetry is fulfilled.

We also examined the sensitivity of the two schemes to detuning of the optical frequency of one laser with respect to the other. We changed the temperature of laser

A while keeping laser *B*'s temperature constant. Variation of the laser temperature shifts the gain curve at a rate of 0.17 nm/K. More important, however, is the shift of internal modes which we measured to be at a rate of 0.043 nm/K. In Fig.6, the effect of detuning on synchronization is shown. Unidirectional coupling (diamonds) is very sensitive to the mismatch of the laser frequencies, as was observed before in ref.[11, 12]. Mutual coupling, on the other hand, is robust and easily accommodates a detuning of ± 2 GHz (± 0.07 K), which corresponds to a significant mismatch in the laser spectra [24]. The detuning of synchronized lasers results in a smaller spectral overlap which effectively reduces the coupling strength σ . For mutually coupled lasers the effective coupling can be varied by a factor of 2 before a significant degradation in synchronization quality is observed (see Fig.3). For unidirectional coupling, however, reduction of effective feedback strength results in a sharp deterioration in the quality of synchronization as shown in Fig.5.

To conclude, we have investigated experimentally the phase-space of feedback and coupling strengths for two mutually coupled lasers each subjected to an external feedback. We have found conditions for which the parameter space for good synchronization is wide. A comparison between unidirectional and mutual coupling, resulting in identical synchronization, revealed that while the first is extremely sensitive to deviations in the feedback strengths and laser frequency detuning, the latter shows robust synchronization even under non identical operation conditions and might be suitable for optical communication applications [9].

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of hundreds of ns while the chaotic features we are interested in are an order of magnitude smaller in intensity and fluctuate on sub-ns time scales. In order to obtain accurate result for the overlap, we divide the intensity traces into 10ns segments (containing 10 sample points) and the correlation coefficient is calculated between matching segments and then averaged over long time stretches (including the intensity breakdowns them-

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